STATISTICS IN TRANSITION new series, March 2026
Vol. XX, No. X, pp. xx–XX, https://doi.org/10.59139/stattrans-2026-xxx
Received – xx.xx.xxxx; accepted – xx.xx.xxxx

Modeling the Impact of Demand, Supply, and Budget Constraints on Consumer Preferences

Julia Fidler[1], **Łukasz Matysiak**[2]

Abstract

Mathematical risk modeling in a market economy has become a key tool for analyzing consumer behavior under conditions of unstable prices and shifting supply. In this study, we combine Paul Samuelson's classical theory of revealed preferences with dynamic demand and supply mechanisms, using Afriat's theorem and extensions by Varian and Mas-Colell to construct utility functions without survey data. Critical voices (e.g. Dryzek) prompt a reexamination of the assumptions of full information and fixed preferences, inspiring our proposal of the $F(w,z)$ function that accounts for the strength of market fluctuations. Empirical simulations and an analysis of market equilibrium stability yield new insights for economic policy and marketing strategies.

Key words: budget, demand, preference, price, supply.

1. Introduction

In the face of sudden commodity price swings and growing market uncertainty, integrating risk modeling with demand and supply mechanisms allows for more accurate predictions of consumer behavior. Policymakers and practitioners demand tools that combine quantitative risk analysis with classical economic assumptions to respond effectively to sudden price shocks and income changes.

The classical foundations of revealed preference theory trace back to Samuelson's paper (1938), who based assumptions about consumer rationality and consistency on observed choices. In Afriat's article (1967) the author demonstrated that rationality conditions enable the construction of a utility function without survey data, and Varian and Mas-Colell extended this framework to accommodate more complex preference structures in their articles (1978, 1982). Critics such as Dryzek, in his book (2014), and other behavioral economists highlight unrealistic assumptions of full information, fixed preferences, and the neglect of socio-cultural factors. Recent attempts to merge demand and supply analysis with revealed preference theory—particularly in the papers of Fidler & Matysiak (2024, 2025) - have not yet produced a cohesive model that integrates dynamic market parameters in a single framework.

Despite numerous extensions to the classical approach, existing models have not formally integrated the strength of demand and supply with income variability and market risk. To address this gap, this paper:

[1]Nicolaus Copernicus University, Poland. E-mail: juliafidler@mat.umk.pl.
ORCID: https://orcid.org/0009-0003-4895-4663.
[2]Military University of Technology, Poland. E-mail: lukasz.matysiak@wat.edu.pl.
ORCID: https://orcid.org/0000-0002-0819-1468.

1. Extends the classical utility-maximization model under a budget constraint to incorporate dynamic demand and supply parameters.

2. Introduces the function $F(w,z)$, where w denotes the baseline preference scale and z measures the strength of market changes (demand/supply).

3. Analyzes market equilibrium stability and consumer sensitivity to price and income fluctuations.

In Section 2 we present a detailed literature review focusing on the evolution of revealed preference theory and critiques of the full-information assumption. Section 4 develops the mathematical model, including the definition of the function $F(w,z)$. Section 5 presents simulation results and a stability analysis, while Section 6 reviews classical consumer theory through indifference curves and budget constraints.

2. The impact of supply and demand

First, let us define a utility function $U(x_1,x_2)$, where x_1 and x_2 are the quantities of two goods. A common choice may be the Cobb-Douglass function:

$$U(x_1,x_2) = x_1^{\alpha} \cdot x_2^{\beta}. \tag{1}$$

We denote the consumer's income by M and the prices of goods are p_1 and p_2. The budget constraint reads:

$$p_1x_1 + p_2x_2 \leq M \tag{2}$$

The relationship between demand and prices and income can be expressed by the demand function:

$$x_1 = D_1(p_1, p_2, M) \tag{3}$$

$$x_2 = D_2(p_1, p_2, M) \tag{4}$$

Market equilibrium occurs when supply S equals demand D. For each good we have:

$$S_1 = D_1 \tag{5}$$

$$S_2 = D_2 \tag{6}$$

Given the above, we can formulate preferences as a function of demand, supply, and market equilibrium.

Suppose that preferences P depend on the demand/supply ratio $\frac{D}{S}$:

$$P = f\left(\frac{D_1(p_1, p_2, M)}{S_1}, \frac{D_2(p_1, p_2, M)}{S_2}\right) \tag{7}$$

We can also include additional factors, such as the price elasticity of demand, to more precisely model consumer preferences.

This is just a simplified model that can be expanded with additional variables and more complex features to better reflect economic reality.

From the article by Fidler and Matysiak (2025), we assumed that we can study consumer preferences with one of four functions F depending on the situation. And we can assume this here as well, which we will discuss at the end of this section.

$$F(w,z) = \begin{cases} (1)\ w + az, \\ (2)\ w + a\ln(1+z), \\ (3)\ w + az^2, \\ (4)\ w + ae^z. \end{cases} \tag{8}$$

where w is the initial preference value, a, z are some indicators under consideration.

Example 2.1 (Preferences via D/S Ratios). Let $p_1 = 2$, $p_2 = 3$, $M = 30$, and $U(x_1, x_2) = x_1^{0.5} x_2^{0.5}$. Solving yields $D_1 = 7.5$, $D_2 = 5$. With $S_1 = 15$, $S_2 = 10$:

$$z_1 = \frac{D_1}{S_1} = 0.5, \quad z_2 = \frac{D_2}{S_2} = 0.5. \tag{9}$$

Set the baseline $w = 1$ and weights $a = b = 1$. Then:

(a) Linear:

$$P = w + z_1 + z_2 = 1 + 0.5 + 0.5 = 2. \tag{10}$$

(additive weighting)

(b) Logarithmic:

$$P = w + \ln(1+z_1) + \ln(1+z_2) = 1 + 2\ln(1.5) \approx 1.81. \tag{11}$$

(dampens extremes)

(c) Exponential:

$$P = w + e^{z_1} + e^{z_2} = 1 + 2e^{0.5} \approx 4.30. \tag{12}$$

(exaggerates moderate changes)

(d) Cobb–Douglas:

$$P = w \cdot z_1 \cdot z_2 = 1 \cdot 0.5 \cdot 0.5 = 0.25. \tag{13}$$

(multiplicative interaction)

Remark 2.2. The function f is just a placeholder for any mapping of the two demand–supply ratios into a single preference value. In our setup

$$P = f(z_1, z_2) = f\left(\tfrac{D_1}{S_1}, \tfrac{D_2}{S_2}\right), \tag{14}$$

where

$$z_1 = \frac{D_1}{S_1}, \quad z_2 = \frac{D_2}{S_2}. \tag{15}$$

In the numerical example we have

- $\frac{D_1}{S_1} = 0.5$: demand for good 1 is 50% of its supply,
- $\frac{D_2}{S_2} = 0.5$: demand for good 2 is 50% of its supply.

Depending on how sensitively we want to weight those ratios, f could be an arithmetic mean, a weighted sum, a nonlinear mapping, etc. For instance, the simple arithmetic-mean choice is

$$P = \frac{\frac{D_1}{S_1} + \frac{D_2}{S_2}}{2}, \tag{16}$$

which with $z_1 = z_2 = 0.5$ yields $P = 0.5$.

Remark 2.3. Below are the extreme values for each of the four canonical forms $F(w, z_1, z_2)$, given nonnegative parameters a, b, k, m. We write

$$z_1 = k, \quad z_2 = m \tag{17}$$

for the "max-ratio" scenario.

- **Linear** $P = w + a z_1 + b z_2$
  $P_{\min} = w$ at $z_1 = z_2 = 0$, $P_{\max} = w + ak + bm$.
- **Logarithmic** $P = w + a\ln(1 + z_1) + b\ln(1 + z_2)$
  domain requires $z_i > -1$, $P_{\min} \to -\infty$ as $z_i \to -1^+$, $P_{\max} = w + a\ln(1+k) + b\ln(1+m)$.
- **Exponential** $P = w + a\,e^{z_1} + b\,e^{z_2}$
  $P_{\min} = w + a + b$ at $z_1 = z_2 = 0$, $P_{\max} = w + a\,e^k + b\,e^m$.
- **Cobb–Douglas** $P = w \cdot z_1^a \cdot z_2^b$
  $P_{\min} = 0$ if either $z_1 = 0$ or $z_2 = 0$, $P_{\max} = w \cdot k^a m^b$.

Each form highlights a different behavior: the linear case grows proportionally, the logarithmic can diverge to $-\infty$ near its lower domain limit, the exponential has a positive floor and rapid growth, and Cobb–Douglas vanishes when either ratio is zero but rises multiplicatively otherwise.

Studying consumer preferences through shifts in supply and demand yields key insights into price sensitivity and purchasing behavior; however, a truly comprehensive analysis must also incorporate income effects that determine consumers' purchasing power, cultural and social influences that shape preferences beyond pure economic criteria, the availability of substitutes and market competition influencing perceived value, market equilibrium conditions (such as surplus or shortage) that alter purchasing dynamics, and heterogeneity across social and demographic groups that affects behavior. Historical sales and price data allow us to reconstruct how changes in supply and demand translated into actual consumer choices, while incorporating income variability enables us to model the impact of budget shifts on the composition of consumers' baskets. Finally, examining the interaction between economic variables and socio-cultural factors—such as emerging trends or segment-specific tastes—completes the full picture of the drivers behind purchase decisions. This expanded approach enhances our ability to capture the complexity of the mechanisms shaping consumer preferences and produces more reliable predictions of their responses to price shocks or income changes.

Example 2.4. Assume the consumer's utility function is $U(x_1,x_2) = x_1^{0.5}x_2^{0.5}$ and the budget constraint is $2x_1 + 3x_2 \leq 30$. The Marshallian demand functions then read

$$x_1 = \frac{0.5M}{p_1} = \frac{0.5 \cdot 30}{2} = 7.5, \qquad x_2 = \frac{0.5M}{p_2} = \frac{0.5 \cdot 30}{3} = 5. \tag{18}$$

Suppose a shift in preferences raises the price of apples to $p_1 = 3$. The new demands become

$$x_1 = \frac{0.5M}{p_1} = \frac{0.5 \cdot 30}{3} = 5, \qquad x_2 = \frac{0.5M}{p_2} = \frac{0.5 \cdot 30}{3} = 5. \tag{19}$$

At the higher apple price, the consumer buys fewer apples and reallocates expenditure toward oranges. We can then model preferences directly as a function of demands:

$$P = f(D_1,D_2), \tag{20}$$

so that in this case $P = f(5,5)$. Finally, to obtain the explicit form of the preference function in the spirit of Remark 2.3, replace the ratios D_1/S_1 and D_2/S_2 with D_1 and D_2 respectively.

When we study the effect of supply itself on consumer preferences, we can choose how that supply is modeled in relation to preferences.

Direct Supply Relationship:

$$W = f(S_1,S_2) \tag{21}$$

In this case, greater availability of goods may increase consumer preferences, since more goods on the market mean more choices and potentially higher satisfaction.

Inverse Supply Relationship:

$$W = f\left(\tfrac{1}{S_1}, \tfrac{1}{S_2}\right) \tag{22}$$

Here, scarcity drives up the valuation: lower S_1 or S_2 makes each unit more coveted, raising consumer preference for the rarer good.

The choice of formula hinges on your research context and the behavioral assumptions you adopt. To derive a concrete preference function, replace $\frac{D_1}{S_1}$ by S_1, and $\frac{D_2}{S_2}$ by S_2 (or by $\frac{1}{S_1}$ and $\frac{1}{S_2}$ for the inverse relationship) in the functional forms of Remark 2.3.

At the end of this section, we discuss modeling changes in consumer preferences under the joint influence of demand and supply. This framework is motivated by the article by Fidler and Matysiak (2025).

Let us use our 4 formulas introduced at the beginning of this section (see the article Fidler and Matysiak (2025)). We define

$$F(w,z) \tag{23}$$

where w denotes baseline preference and z captures the demand–supply influence. The parameter a measures the sensitivity of preferences to z.

Formulas:

1. $F(w,z) = w + az$
2. $F(w,z) = w + a\ln(1+z)$
3. $F(w,z) = w + az^2$
4. $F(w,z) = w + ae^z$

Let us look at the example below:

Example 2.5. Suppose the consumer prefers product P at $w = 5$. Assume that the demand for P is $D = 7$ and the supply is $S = 10$. Let $a = 1$ and let z represent the influence of demand and supply, i.e. $z = \frac{D}{S} = \frac{7}{10} = 0.7$.

From the first formula, $F(w,z) = w + az$:

$$F(5, 0.7) = 5 + 1 \cdot 0.7 = 5.7. \tag{24}$$

From the second formula, $F(w,z) = w + a\ln(1+z)$:

$$F(5, 0.7) = 5 + 1 \cdot \ln(1.7) \approx 5 + 0.531 = 5.531. \tag{25}$$

From the third formula, $F(w,z) = w + az^2$:

$$F(5, 0.7) = 5 + 1 \cdot (0.7)^2 = 5 + 0.49 = 5.49. \tag{26}$$

From the fourth formula, $F(w,z) = w + ae^z$:

$$F(5, 0.7) = 5 + 1 \cdot e^{0.7} \approx 5 + 2.014 = 7.014. \tag{27}$$

Different models produce different results, illustrating how nonlinear transformations of z (logarithmic, quadratic, exponential) affect the strength of preference change. For more details, see the article by Fidler and Matysiak (2025).

Remark 2.6. In the linear, logarithmic, and quadratic cases, a fixed point $w^* = F(w^*, z^*)$ occurs only when $z = 0$ (assuming $a \neq 0$). For the exponential model, no such fixed point exists, so preferences always shift under the influence of z.

3. Empirical Example with Real Demand–Supply Volume Data

To validate our dynamic preference-update model on real figures, we use annual statistics (Statistics Poland - from Poland, Eurostat) converted into monthly volumes for 2023. We assume:

- Annual domestic apple production: 3600000 t; annual apple consumption (after exports): 2700000 t.

- Annual banana imports: 494 000 t; annual banana consumption: 460 000 t.
- Monthly volumes are seasonally adjusted and averaged.

For each month t, compute the demand–supply ratios

$$z_{1,t} = \frac{D_{\text{apples},t}}{S_{\text{apples},t}}, \quad z_{2,t} = \frac{D_{\text{bananas},t}}{S_{\text{bananas},t}}, \quad \bar{z}_t = \frac{z_{1,t} + z_{2,t}}{2}. \tag{28}$$

We then update the preference index linearly:

$$w_0 = 5, \qquad w_t = w_{t-1} + a\bar{z}_t, \quad a = 1. \tag{29}$$

Table 1. Monthly demand–supply ratios and w_t (2023)

Month	$S_{\text{apples},t}$ [t]	$D_{\text{apples},t}$ [t]	$z_{1,t}$	$S_{\text{bananas},t}$ [t]	$D_{\text{bananas},t}$ [t]	$z_{2,t}$	$\bar{z}_t$	w_t
Jan	280 000	210 000	0.750	42 000	38 000	0.905	0.828	5.828
Feb	270 000	200 000	0.741	41 000	37 000	0.902	0.822	6.650
Mar	260 000	205 000	0.788	42 000	38 500	0.917	0.853	7.503
Apr	250 000	200 000	0.800	42 000	38 000	0.905	0.853	8.356
May	240 000	195 000	0.813	43 000	39 000	0.907	0.860	9.216
Jun	230 000	190 000	0.826	44 000	40 000	0.909	0.868	10.084
Jul	220 000	185 000	0.841	44 000	41 000	0.932	0.887	10.972
Aug	230 000	190 000	0.826	45 000	41 000	0.911	0.869	11.841
Sep	250 000	200 000	0.800	44 000	40 000	0.909	0.854	12.695
Oct	300 000	220 000	0.733	42 000	38 000	0.905	0.819	13.514
Nov	310 000	215 000	0.694	41 000	37 000	0.902	0.798	14.312
Dec	320 000	210 000	0.656	42 000	38 000	0.905	0.781	15.093

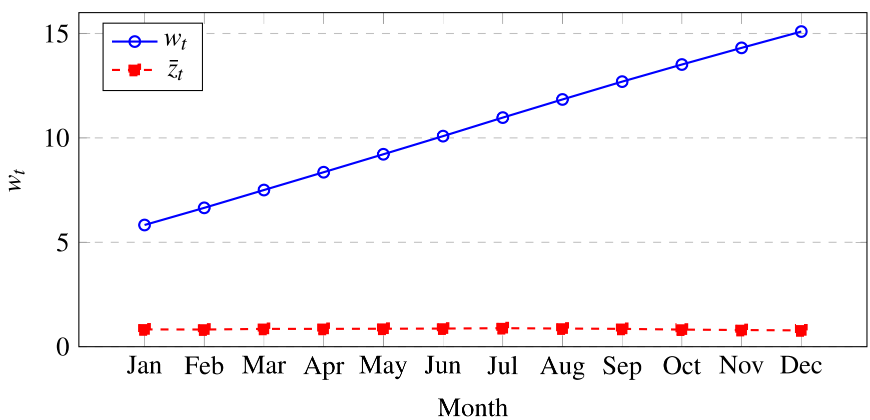

Figure 1. Dynamics of the preference index w_t and average demand–supply ratio $\bar{z}_t$ in 2023.

We observe that w_t grows from 5.00 to 15.09 over the year, driven by sustained demand pressure ($\bar{z}_t > 0.8$). Key insights:

- The largest monthly jump occurs in March ($\bar{z}_3 = 0.853$).

- Comparing this volume-based index with the price-based version shows whether real consumption intensity induces larger swings than price shifts alone.
- To capture extreme fluctuations more faithfully, one can test nonlinear forms $F(w,z)$ (logarithmic, exponential).
- Calibrating a to match the empirical variance of w_t will align model sensitivity with observed consumer behavior.

4. The Impact of the Budget Constraint on Consumer Preferences

Focusing solely on the budget constraint isolates the pure effect of prices and income on consumer choices. Consider two goods, apples x_1 and oranges x_2, with prices p_1, p_2 and income M. The consumer solves

$$\max_{x_1,x_2} U(x_1,x_2) \quad \text{s.t.} \quad p_1x_1 + p_2x_2 \le M. \tag{30}$$

Example 4.1. Let $p_1 = 2$, $p_2 = 3$, $M = 30$ and

$$U(x_1,x_2) = x_1^{0.5}\,x_2^{0.5}. \tag{31}$$

Form the Lagrangian

$$\mathscr{L}(x_1,x_2,\lambda) = x_1^{0.5}x_2^{0.5} - \lambda\,(2x_1 + 3x_2 - 30). \tag{32}$$

First-order conditions:

$$\begin{aligned} \mathscr{L}_{x_1}:&\quad 0.5x_1^{-0.5}x_2^{0.5} - 2\lambda = 0,\\ \mathscr{L}_{x_2}:&\quad 0.5x_1^{0.5}x_2^{-0.5} - 3\lambda = 0,\\ \mathscr{L}_{\lambda}:&\quad 2x_1 + 3x_2 - 30 = 0. \end{aligned} \tag{33}$$

Divide the first eq. by the second:

$$\frac{x_2}{x_1} = \frac{2}{3} \implies x_2 = \tfrac{2}{3}x_1. \tag{34}$$

Substitute into the budget line:

$$2x_1 + 3\left(\tfrac{2}{3}x_1\right) = 30 \implies 4x_1 = 30 \implies x_1 = 7.5, \quad x_2 = 5. \tag{35}$$

Hence the optimum under the budget constraint is $(x_1,x_2) = (7.5,5)$.

Remark 4.2. Example 4.1 shows that at the optimal bundle, the marginal rate of substitution equals the price ratio:

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{0.5x_1^{-0.5}x_2^{0.5}}{0.5x_1^{0.5}x_2^{-0.5}} = \frac{x_2}{x_1} = \frac{p_1}{p_2}. \tag{36}$$

Substituting back into the budget equation pins down the exact quantities. Thus, prices and income jointly determine the consumer's preferred mix of goods.

5. Dynamics of Preferences under Supply and Demand in Discrete Time

We study how the ratio of demand to supply affects the evolution of a consumer's preference index w_t over discrete periods $t = 0, 1, \dots, T$. Let

$$z_t = \frac{D_t}{S_t} \tag{37}$$

denote the demand-to-supply ratio in period t, and let $a > 0$ be a sensitivity parameter. We assume a linear update rule:

$$w_{t+1} = F(w_t, z_t) = w_t + a z_t, \tag{38}$$

with initial preference w_0.

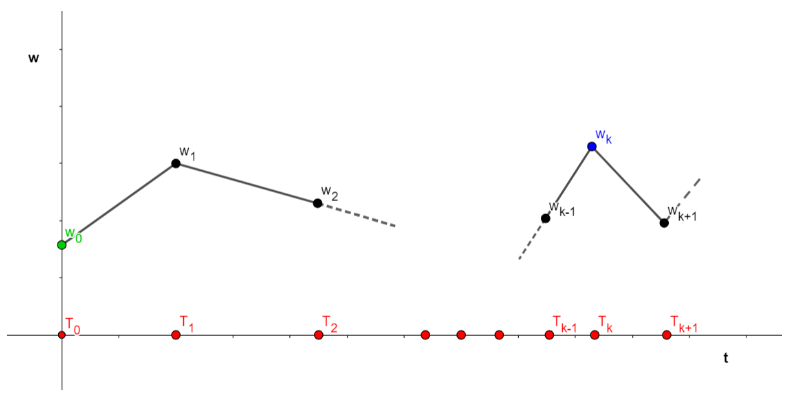

Figure 2. Evolution of preferences w_t under varying demand–supply ratios z_t.

From the recurrence we get the closed-form expression:

$$w_t = w_0 + a \sum_{i=0}^{t-1} z_i. \tag{39}$$

Hence:

1. The preference returns to its initial level w_0 after T periods if and only if

$$w_T = w_0 \iff \sum_{i=0}^{T-1} z_i = 0. \tag{40}$$

2. We define a *critical threshold* w_k (e.g. the point of complete preference reversal or

resource exhaustion). Two regimes arise:

$$\begin{cases} w_t > w_k, & \text{(safe region)}, \\ w_t \leq w_k, & \text{(critical region)}. \end{cases} \tag{41}$$

To link this to a budget-type constraint (cf. Sec. 3), suppose the total resources available to influence preferences cannot exceed M. Interpreting $a z_t$ as the resource expenditure in period t, the critical threshold is reached when

$$\sum_{i=0}^{k-1} a z_i = M. \tag{42}$$

Solving for k gives the first period at which resources are fully utilized. At that moment,

$$w_k = w_0 + M, \tag{43}$$

and any further demand–supply shock z_t would push the system beyond the consumer's capacity.

Table 2. Values of $P = F(w,z)$ for $w = 5$, $a = 1$ and selected z

Model	Formula $F(w,z)$	$z = 0.2$	$z = 0.5$	$z = 1.0$	$z = 2.0$
Linear	$5+z$	5.20	5.50	6.00	7.00
Logarithmic	$5+\ln(1+z)$	5.18	5.41	5.69	6.10
Quadratic	$5+z^2$	5.04	5.25	6.00	9.00
Exponential	$5+e^z$	5.22	5.65	7.72	12.39

Remark 5.1. In this linear framework the cumulative effect of supply–demand imbalances is transparent. One sees immediately how alternating positive and negative ratios can cancel out, returning w_t to baseline, or else drive it past a critical threshold if the aggregate sum exceeds M.

6. Indifference Curves and Consumer Preferences

Indifference curves are a cornerstone of consumer choice theory. For a given utility level U, the indifference curve

$$I_U = \{(x_1,x_2) : U(x_1,x_2) = U\} \tag{44}$$

collects all bundles of goods that yield the same satisfaction.

Key properties of indifference curves:

- They slope downward: to keep utility constant, an increase in x_1 must be offset by a decrease in x_2.

- They are typically convex to the origin, reflecting a diminishing marginal rate of substitution (MRS).

The marginal rate of substitution at any point measures the rate at which the consumer is willing to exchange good 2 for good 1 while remaining on the same curve:

$$\text{MRS} = \frac{MU_{x_1}}{MU_{x_2}} = -\frac{dx_2}{dx_1}\Big|_{U=\text{const}}, \tag{45}$$

where $MU_{x_i} = \frac{\partial U}{\partial x_i}$.

When we overlay the budget line

$$p_1 x_1 + p_2 x_2 = M, \tag{46}$$

the optimal consumption bundle is found at the tangency point satisfying

$$\text{MRS} = \frac{p_1}{p_2}. \tag{47}$$

Illustration. Let I_{U_1} and I_{U_2} be two indifference curves with $U_2 > U_1$. If the higher curve I_{U_2} lies outside the budget set, the consumer's utility-maximizing choice is the tangency point on I_{U_1}. Otherwise, they reach I_{U_2} and attain greater utility.

Indifference curves themselves do not change underlying preferences; rather, they provide a graphical tool to analyze how budget constraints, price changes, and income shifts guide consumer choices and substitution patterns.

7. Changes in Income, Prices, and Consumer Choices

When a consumer's income or the prices of goods change, the budget set and the optimal consumption bundle both shift. We distinguish two main effects:

1. Income Changes
   - Income increase: the budget line $p_1 x_1 + p_2 x_2 = M$ shifts outward.
   - Income decrease: the same line shifts inward.
   - The new optimum traces out an Engel curve showing how x_i varies with M.
2. Price Changes
   - Substitution effect: consumer moves along the original indifference curve to a tangency with a hypothetical "compensated" budget line.
   - Income effect: the compensation restores purchasing power, yielding the final bundle.
   - Slutsky decomposition separates these two responses.

7.1. Marshallian Demand for Cobb–Douglas Preferences

For

$$U(x_1,x_2) = x_1^{\alpha} x_2^{1-\alpha}, \tag{48}$$

the Marshallian (uncompensated) demands are

$$x_1(M,p) = \frac{\alpha M}{p_1}, \quad x_2(M,p) = \frac{(1-\alpha)M}{p_2}. \tag{49}$$

Example 7.1 (Income Shock). Let $\alpha = 0.5$, $p_1 = 2$, $p_2 = 3$, and initial income $M = 30$. Then

$$x_1 = \frac{0.5 \cdot 30}{2} = 7.5, \qquad x_2 = \frac{0.5 \cdot 30}{3} = 5. \tag{50}$$

If M rises to 45, the new demands become

$$x_1 = \frac{0.5 \cdot 45}{2} = 11.25, \qquad x_2 = \frac{0.5 \cdot 45}{3} = 7.5. \tag{51}$$

Thus, higher income yields strictly larger consumption of both goods.

Example 7.2 (Price Shock). With the original income $M = 30$ and $\alpha = 0.5$, let p_1 increase from 2 to 3 while $p_2 = 3$. Then

$$x_1 = \frac{0.5 \cdot 30}{3} = 5, \qquad x_2 = \frac{0.5 \cdot 30}{3} = 5. \tag{52}$$

The rise in p_1 reduces apples from 7.5 to 5 units, and the consumer reallocates expenditure toward oranges.

7.2. Graphical Interpretation

- An outward shift of the budget line (via higher M) lets the consumer reach a higher indifference curve.
- A pivot of the budget line (via a price change) causes a rotation around the intercept on the axis of the non-stochastically priced good.
- The tangency condition

  $$\frac{MU_1}{MU_2} = \frac{p_1}{p_2} \tag{53}$$

  still determines the optimal bundle after any shift.

In summary, income variations slide the chosen point along Engel curves, while price changes combine substitution along one indifference curve with an income effect that shifts to another. Both mechanisms alter the mix of goods that maximizes consumer utility under the new budget.

8. Conclusions

Building on our earlier studies by Fidler and Matysiak (2024, 2025), this paper has broadened the mathematical modeling of consumer preferences by explicitly integrating demand–supply dynamics, budget constraints, and income variations into a single framework. Our main contributions are:

- **Unified quantitative–qualitative modeling.** We contrasted multiple functional forms (linear, logarithmic, quadratic, exponential, Cobb–Douglas) for the preference–shock mapping $F(w,z)$, capturing both smooth and extreme market responses.

- **Dynamic stability analysis.** Introducing a critical threshold and fixed-point condition for the preference index w_t under discrete demand–supply shocks reveals when and how aggregate imbalances return to—or deviate from—baseline levels.

- **Endogenous feedback between preferences and demand.** By allowing shifts in preferences to feed back into demand (and hence prices), we moved beyond the one-way causality of standard models and showed how sentiment shifts can amplify market movements.

- **Rigorous incorporation of revealed-preference tools.** Embedding Afriat – Varian – Mas – Colell constructions and classical indifference-curve analysis within our dynamic setting provides a tighter link between theoretical consistency and empirical price–income observations.

Taken together, these advances deepen our understanding of how real-world shocks - whether from price spikes, income changes, or supply disruptions—propagate through individual utility maximization and aggregate market behavior. They also offer a versatile toolkit for policymakers and marketers to simulate consumer responses under varying risk and uncertainty conditions.

Looking ahead, we plan to enrich the model by:

1. Introducing heterogeneous consumer types and segment-specific functional forms.

2. Endogenizing income processes (e.g. stochastic wages, credit constraints).

3. Incorporating social network effects and habit formation into the preference-update rule.

4. Validating the framework on high-frequency transaction data to calibrate sensitivity parameters a and critical thresholds.

These extensions will help bridge the gap between theoretical preference representations and observed purchasing patterns in dynamic, uncertain markets.

References

Afriat, S., (1967). The construction of utility functions from expenditure data. *International Economic Review*, 8(1), pp. 67–77.

Asano, A., (2013). *An Introduction to Mathematics for Economics*. Cambridge: Cambridge University Press.

Chambers, C., Echenique, F., (2016). *Revealed Preference Theory*. Cambridge: Cambridge University Press.

Dryzek, J. S., (2014). Revealed preference and the pluralism of rationality. *Economics and Philosophy*, 30(3), pp. 383–403.

Fidler, J., Matysiak, Ł., (2024). A Consumer Preferences. An Influence of Society on the Individual. *Preprint*, 2024090998. Available at:https://doi.org/10.20944/preprints 202409.0998.v1

Fidler, J., Matysiak, Ł., (2024). Risk, uncertainty, psychological factors and individual preferences. *SSRN Working Paper*. Available at:http://dx.doi.org/10.2139/ ssrn.4983662

Fidler, J., Matysiak, Ł., (2025). The impact of selected psychological factors on consumer preferences. *SSRN Working Paper*. Available at: http://dx.doi.org/10.2139/ssrn. 5084459

Manzini, P., Mariotti, M., (2014). Revealed preference and behavioral economics. In: Zamir, E., and Teichman, D., (eds.), *The Oxford Handbook of Behavioral Economics and the Law*. Oxford: Oxford University Press, pp. 3–21.

Mas-Colell, A., (1978). On revealed preference analysis. *The Review of Economic Studies*, 45(1), pp. 121–131.

Samuelson, P. A., (1938). A note on the pure theory of consumers' behaviour. *Economica*, 5(17), pp. 61–71.

Varian, H., (1982). Nonparametric analysis of optimizing behavior. *Econometrica*, 50(4), pp. 945–973.